

An Adaptive Hybrid Algorithm for Time Series Prediction in Healthcare

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Abstract – Prediction models based on different concepts have been proposed in recent years. The accuracy rates resulting from linear models such as exponential smoothing, linear regression (LR) and autoregressive integrated moving average (ARIMA) are not high as they are poor in handling the nonlinear relationships among the data. Neural network models are considered to be better in handling such nonlinear relationships. Healthcare time series data such as Morbidity of Tuberculosis (MTB) consist of complex linear and nonlinear patterns and it may be difficult to obtain high prediction accuracy rates using only linear or neural network models. Hybrid models which combine both linear and neural network models can be used to obtain high prediction accuracy rates. In this paper, we propose an adaptive hybrid algorithm to achieve the best results for time series prediction in healthcare. We also make a comparison of the proposed model with other known models based on accuracy rates.

Keywords: Adaptive Hybrid Algorithm, Neural Network, Exponential Smoothing, Linear Regression, ARIMA

I. INTRODUCTION

Many prediction models have been proposed recently for improving accuracies in time series prediction. The accuracy of a forecasting method would depend not only on the model but also on the complexity of the data. Hence, it is important to choose the best model based on the complexity of data in time series prediction.

In the real-world, the time series data often contain linear and nonlinear patterns. Linear models have limitations in handling the nonlinear relationships among the data. Neural network models are considered to be better in handling such nonlinear relationships, but they may not be efficient to deal with the linear pattern.

Many models have been applied in time series forecasting such as autoregressive integrated moving average (ARIMA) [1, 2], linear regression (LR)[3, 4] and neural network [5, 6, 7]. Hybrid models combining ARIMA

and neural network models have also been used for some applications [8].

In this paper, we propose an adaptive hybrid algorithm which selects the appropriate combination of linear nonlinear models depending on the complexity of data. The proposed model is applied for a specific application, namely, prediction of morbidity of tuberculosis (MTB). Prediction of MTB assumes importance in healthcare management as it helps in devising appropriate action plans. The MTB data consist of complex linear and nonlinear patterns and it may be difficult to obtain high prediction accuracy rates using only linear or nonlinear models. An adaptive hybrid algorithm is proposed in this paper which can be used for predicting complex time series data such as MTB. Depending on a linearity test, the hybrid algorithm will select either a linear-nonlinear combination or a nonlinear-linear combination. The performances of four linear models, namely, exponential smoothing (single and double), linear regression (LR) and autoregressive integrated moving average (ARIMA) models are compared initially and the best model is selected based on the performance measure. For the nonlinear model, we use multi-layer perceptron neural network (MLP).

The paper is organized as follows. In the next section, we review different models used such as exponential smoothing, linear regression, ARIMA, neural network and hybrid models. In section 3, we present the adaptive hybrid algorithm for time series prediction. Section 4 describes the data used for simulation experiments. The performance measures of the different models and also a comparison of these results are presented in Section 5. Section 6 contains the concluding remarks.

II. MODELS USED

This section describes the linear and nonlinear models implemented in this paper. Linear models consist of Single and Double Exponential Smoothing, Linear Regression and ARIMA. The nonlinear model makes use of MLP neural network.

A. Linear Models

1) Exponential Smoothing Model

The exponential smoothing model assumes that the data are closer to the current data, which is considered to be more important for predicting the future data. There are several types of the models that are used in exponential smoothing, such as single exponential smoothing and double exponential smoothing.

The equation of single exponential smoothing model is given:[9]

$$\hat{y}_t = \alpha y_t + (1 - \alpha) \hat{y}_{t-1} \quad (1)$$

where α is the smoothing constant.

And the equation of double exponential smoothing model is given: [9]

$$\begin{aligned} L_t &= \alpha y_t + (1 - \alpha)(L_{t-1} + T_{t-1}) \\ T_t &= \beta (L_t - L_{t-1}) + (1 - \beta)T_{t-1} \\ \hat{y}_{t+p} &= L_t + pT_t \end{aligned} \quad (2)$$

where α and β are the smoothing constants, $L_0 = y_1$ and $T_0 = 0$.

y_t : Actual data at time $t, t=1, \dots, n$

L, T : The level and trend estimate at time t

\hat{y}_{t+p} : The forecast (prediction) of time $t+p$ made at time t and p periods into the future.

2) Linear Regression Model

Linear regression model is one of the most commonly used methods for forecasting. The regression model describes the mean of the normally distributed dependent variable y as a function of the predictor or independent variable x [10]:

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i \quad (3)$$

where y_i is the value of the response or dependent variable from the i -th pair, β_0 and β_1 are the two unknown parameters, x_i is the value of the independent variable from the i -th pair, and ε_i is a random error term.

The predicted or estimated or fitted values of the regression model are calculated as [11]:

$$\hat{y}_i = b_0 + b_1 x_i \quad (4)$$

The parameters b_0 and b_1 in Eq. (4) are computed as [12]:

$$b_0 = \frac{\sum_{i=1}^n y_i}{n} - b_1 \frac{\sum_{i=1}^n x_i}{n} \quad (5)$$

$$b_1 = \frac{n \sum_{i=1}^n x_i y_i - \sum_{i=1}^n x_i \sum_{i=1}^n y_i}{n \sum_{i=1}^n x_i^2 - (\sum_{i=1}^n x_i)^2}$$

This model is referred to as the linear regression (LR) model. It is linear because the independent variable appears only in the first power; if we plot the mean of Y versus x , the graph will be a straight line with intercept b_0 and slope b_1 .

3) ARIMA Model

Time series analysis has several objectives such as modeling, forecasting and controlling. Forecasting deals with the issue of constructing models and methods that can be used to produce accurate short-term predictions [13].

The forecast in autoregressive (AR) model is a function of its past observations (x), and the forecast in a moving average (MA) model is a function of its past residuals (e). An autoregressive model of order p is given by:

$$\hat{x}_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + \dots + \phi_p x_{t-p} \quad (6)$$

and a moving average model of order q is given by

$$\hat{x}_t = \theta_1 e_{t-1} + \theta_2 e_{t-2} + \dots + \theta_q e_{t-q} \quad (7)$$

The order of an autoregressive polynomial is denoted by p and that of a moving average polynomial is denoted by q .

An autoregressive moving average models are made up autoregressive (AR(p)) part and a moving average (MA (q)) part. An ARMA (p, q) model is defined by [14]:

$$x_t = \sum_{i=1}^p \phi_i x_{t-i} + e_t + \sum_{j=1}^q \theta_j e_{t-j} \quad (8)$$

where e_t is the random error at time t , ϕ_i ($i=1, 2, \dots, p$) and θ_j ($j=1, 2, \dots, q$) are the model parameters to be estimated. The orders of autoregressive and moving average polynomials are p and q respectively. The backshift operator (B) is used to make the notational form simple. The backshift operator is defined by:

$$B^j x_t = x_{t-j} \quad (9)$$

where $j=0, 1, 2, \dots$

An autoregressive moving average of order p and q , ARMA (p, q) model can be expressed as

$$\phi(B)x_t = \theta(B)e_t \quad (10)$$

where $\phi(\cdot)$ and $\theta(\cdot)$ are the p^{th} and q^{th} degree polynomials are given by [14]:

$$\phi(z) = 1 - \phi_1 z - \phi_2 z^2 - \dots - \phi_p z^p \quad (11)$$

$$\theta(z) = 1 + \theta_1 z + \theta_2 z^2 + \dots + \theta_q z^q$$

An autoregressive moving average (ARMA) models employed for time series data using ordinary differencing are called autoregressive integrated moving average (ARIMA) models. An autoregressive integrated moving average model with order of differencing d , ARIMA (p, d, q) can be expressed as [14]:

$$\phi(B)(1-B)^d x_t = \theta(B)e_t \quad (12)$$

where B represents the backshift operator.

B. Neural Network Model

In recent years, neural networks (NN) for forecasting have been investigated by many researchers. The motivation for using neural networks is that these models are capable of handling nonlinear relationships.

The MLP neural network model consists of different layers which are connected to each other by connection weights. Between the extremities of the input and the output layer are the hidden layers. The nodes in each layer are connected by flexible weights, which are adjusted based on the error or bias.

The function of the input layer is for data entry, data processing takes place in the hidden layer and the output layer functions as the data output result. Fig. 1 shows the architecture of a general multilayer perceptron neural network.

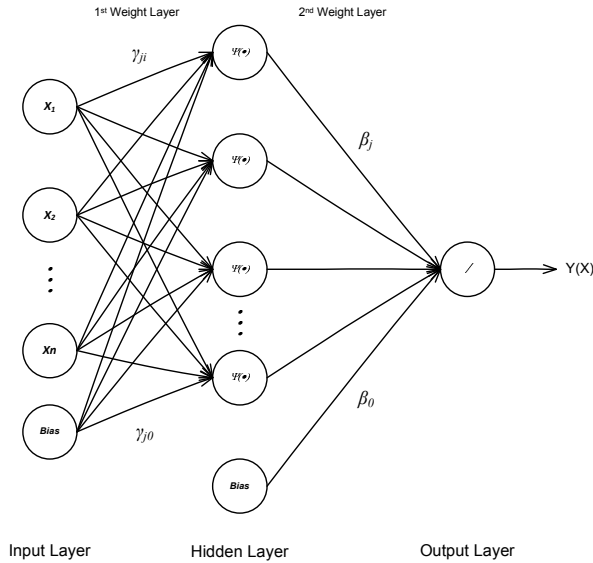


Figure 1. A General Multilayer Perceptron Neural Network

In this architecture, the response value $Y(x)$ is computed as [15]:

$$Y(x) = \beta_0 + \sum_{j=1}^H \beta_j \psi(\gamma_{j0} + \sum_{i=1}^n \gamma_{ji} x_i) \quad (13)$$

where $(\beta_0, \beta_1, \dots, \beta_H, \gamma_{10}, \dots, \gamma_{Hn})$ are the weights or parameters of the neural network. The non linearity enters into the function $Y(x)$ through the so called activation function ψ .

Processing in each neuron is done with summing the multiplication results of connection weights by the input data. The result will be transferred to the next neuron via activation function.

In this paper, we use several kinds of activation functions ψ , such as sigmoid, bipolar sigmoid and hyperbolic tangent.

The Hyperbolic Tangent activation function [16] given below:

$$\psi(x_j) = \tanh(x_j) = \frac{e^{x_j} - e^{-x_j}}{e^{x_j} + e^{-x_j}} \quad (14)$$

C. Hybrid Model

Linear and neural network models have achieved successes in their linear and nonlinear domains respectively. However, the use of linear models for complex nonlinear problems may not yield accurate results. On the other hand, using neural network models for linear problems has yielded mixed results [17]. Since it is difficult to know the characteristics of the data in a real problem, hybrid methodology that has both linear and nonlinear modeling capabilities can be a good strategy for obtaining accurate prediction.

In general, a time series data is composed of a linear autocorrelation structure and a non-linear component as shown in Eq. (14) [17],

$$y_t = L_t + N_t \quad (14)$$

where L_t and N_t represent the linear and nonlinear components respectively.

Hybrid models using a combination of linear methods (such as ARIMA) and nonlinear methods (such as NN) have been proposed recently [17, 18].

D. Performance Measures

There exist several performance measures to calculate the prediction efficiency. In this paper, we employ measures such as root mean square error (RMSE), mean absolute error (MAE), and mean absolute percentage error (MAPE).

1) Root Mean Square Error (RMSE)

The RMSE is computed as [8,19]:

$$RMSE = \sqrt{\frac{\sum_{t=1}^n (Y_t - \hat{Y}_t)^2}{n}} \quad (15)$$

2) Mean Absolute Error (MAE)

The mean absolute error (MAE) is computed as [19]:

$$MAE = \frac{\sum_{t=1}^n |Y_t - \hat{Y}_t|}{n} \quad (16)$$

3) Mean Absolute Percentage Error (MAPE)

MAPE produces a measure of relative overall fit [8]:

$$MAPE = \frac{\sum_{t=1}^n \frac{|Y_t - \hat{Y}_t|}{Y_t} * 100}{n} \quad (17)$$

III. THE ADAPTIVE HYBRID FORECASTING

The proposed adaptive hybrid algorithm is illustrated in Fig. 2. Depending on the result of a linearity test, either a linear-nonlinear combination or a nonlinear-linear combination will be selected as shown in Fig.2. The algorithm selects the best linear model after comparing the performance results of four linear models, namely,

exponential smoothing (single and double), linear regression (LR) and autoregressive integrated moving average (ARIMA) models.

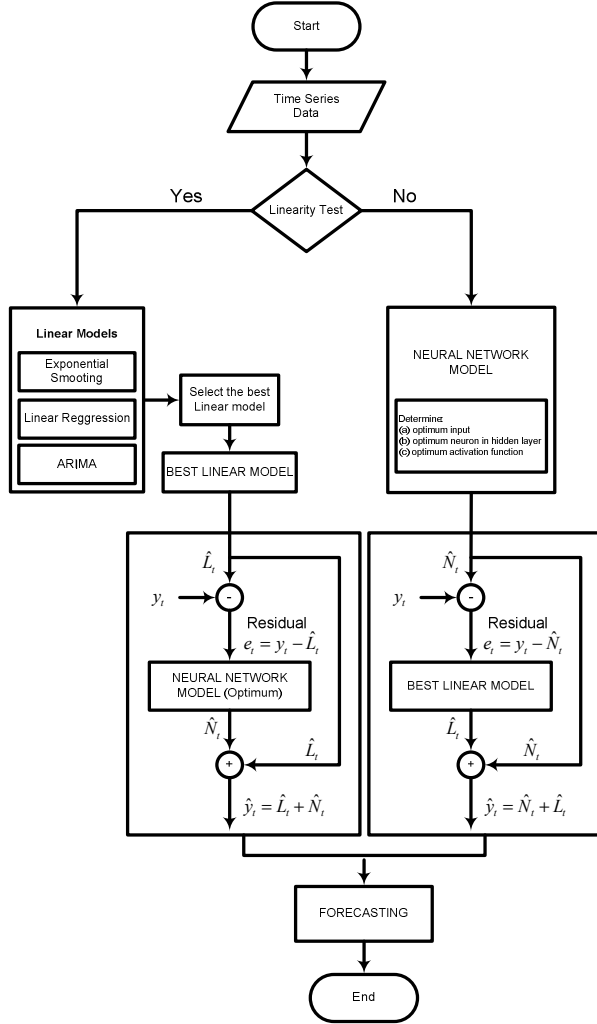


Figure 2. Flowchart of Adaptive hybrid for Forecasting

We use Ramsey RESET test (regression equation specification error test) method for testing the linearity[20]. If the linearity test yields a positive result, a linear-nonlinear combination is selected in which, a linear model is first used to predict the linear component \hat{L}_t . The residual or error series e_t obtained from the linear model contains information on the non-linearity of the series,

$$e_t = y_t - \hat{L}_t \quad (18)$$

e_t is then applied to a nonlinear model using MLP with optimum configuration to obtain the predicted output \hat{N}_t , which is combined with \hat{L}_t to get the overall prediction of the hybrid model as shown in Eq. (19),

$$\hat{y}_t = \hat{L}_t + \hat{N}_t \quad (19)$$

where \hat{y}_t represents the combined forecast value of the hybrid model at time t .

On the other hand, if the linearity test yields a negative result, a nonlinear-linear combination will be used. The steps involved in this process are illustrated in Fig.2.

IV. DATA

The healthcare data such as morbidity used in this study are collected in Indonesia from Indonesian Health Profile, World Health Organization (WHO), etc [21, 22, 23].

Morbidity is the total number of people who suffer a certain disease, such as Malaria, Tuberculosis. In this paper, we use morbidity of tuberculosis per 100,000 populations during the period 1990-2008 in Indonesia.

The descriptive statistics of the MTB data such as the minimum, maximum, mean, standard deviation and Variance are shown in Table 1.

Table 1. Descriptive Statistics of MTB in Indonesia

Name	Min	Max	Mean	Std. Dev.	Variance
MTB	244.2	442.8	336.1	64.5	4161.8

V. EXPERIMENT STUDY

In this section, we compute the prediction results based on the proposed adaptive hybrid algorithm. For linearity test, we applied Ramsey RESET test. For the MTB data the linearity test yielded a positive result and hence the linear-nonlinear combination was selected.

To select the best linear model and the optimum MLP network, the performance measures (MAE, RMSE and MAPE) achieved by each implemented model are discussed below.

A. Exponential Smoothing

In this paper, we use single exponential smoothing and double exponential smoothing. The results of performance measures using the models are shown in Table 2. We compute prediction using single exponential smoothing models with different weight of smoothing (α).

Table 2. Performance Measures for Exponential Smoothing

MODELS	PERFORMANCE MEASURES			
	MAE	RMSE	MAPE	
SINGLE EXPONENTIAL SMOOTHING (α)	0.10	54.46	61.44	18.34
	0.20	39.27	42.49	12.84
	0.40	24.31	25.70	7.62
	0.60	17.65	18.86	5.42
	0.80	14.56	15.42	4.40
	1.00	12.59	13.54	3.75
	1.20	11.22	12.58	3.31
	1.50	10.00	12.56	2.90
	1.60	10.15	13.06	2.90
1.80	12.46	16.12	3.45	
DOUBLE EXPONENTIAL SMOOTHING	3.27	6.09	1.14	

B. Linear Regression

The equation of Linear Regression model for MTB time series forecasting is obtained as follows:

$$y_t = 450.28 - 11.4219 * t \quad (20)$$

where y_t is the predicted value for t -th year.

The result of the performance measures with Linear Regression model is shown in Table 3.

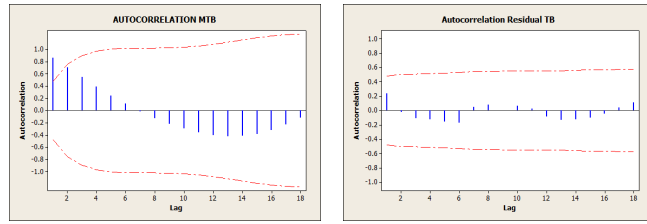
Table 3. Performance Measures for Linear Regression

PERFORMANCE MEASURES	VALUES
MAE	3.16
MAPE	1.11
RMSE	5.38

C. ARIMA Models

ARIMA models assume that the data are stationary. If data are not stationary, they are made stationary by performing differencing.

We compute autocorrelation of the MTB data to check whether the data are stationary.



(a). preliminary data (b) After differencing one time

Fig 3. Autocorrelation Function for MTB data

From Fig. 3 (a), it is seen that there is a blue bar that goes beyond the red line, indicating that differencing process needs to be done. Autocorrelation function for MTB data after performing differencing process one time is shown in Fig. 3(b) and there is no a blue bar that goes beyond the red line (stationary data).

Subsequently, we compute prediction using ARIMA models with different parameter (p, d, q) values with d equal to 1. The result of the performance measures with different ARIMA models is shown in Table 4. Testing with other parameter values was also undertaken but not shown in the table.

Table 4. Performance Measures for ARIMA Models

MODELS	PERFORMANCE MEASURES		
	MAE	RMSE	MAPE
ARIMA(0,1,1)	2.77	4.48	0.94
ARIMA(0,1,2)	54.58	64.51	17.03
ARIMA(1,1,0)	3.03	4.86	1.04
ARIMA(1,1,2)	2.89	4.67	0.97
ARIMA(2,1,0)	2.89	4.77	0.99
ARIMA(2,1,1)	2.57	4.51	0.90
ARIMA(2,1,2)	2.55	4.43	0.90
ARIMA(2,1,3)	2.59	4.68	0.92
ARIMA(3,1,1)	2.91	4.67	1.00

D. Neural Network Model

In this paper, we use multi-layer perceptron neural network model for nonlinear model. Architecture configurations with different values of input and hidden layer neurons were tested to determine the optimum configuration. Similarly, different activation functions such as hyperbolic tangent, bipolar sigmoid and sigmoid functions were also tested. From the experimental results, it is found that the neural network model with 5 input neurons, 10 hidden layer neurons and using hyperbolic tangent activation functions for the hidden and output layers yields the minimum values for MAE, MAPE and RMSE.

Table 5. Performance Measures for Neural Network Models

MODELS	Activation Function	PERFORMANCE MEASURES		
		MAE	RMSE	MAPE
NN(1,10,1)	Hip. Tangent	2.99	3.51	1.07
NN(2,10,1)	Hip. Tangent	3.21	3.90	1.18
NN(3,10,1)	Hip. Tangent	2.63	3.22	0.96
NN(4,10,1)	Hip. Tangent	2.94	3.52	1.05
NN(5,10,1)	Hip. Tangent	2.22	2.84	0.85
NN(5,10,1)	Bipolar Sigmoid	4.22	4.85	1.47
NN(5,10,1)	Sigmoid	8.94	10.12	2.82
NN(6,10,1)	Hip. Tangent	2.54	3.18	0.93
NN(6,10,1)	Bipolar Sigmoid	3.43	3.95	1.22
NN(6,10,1)	Sigmoid	6.54	7.30	2.25

The results of performance measures using different configurations of neural network model are shown in Table 5.

E. Hybrid Model

To select the best linear model, a comparison of performance measures of the different linear models is shown Table 6.

Table 6. Comparison of Performance Measures for Linear Models

NO	MODELS	PERFORMANCE MEASURES		
		MAE	RMSE	MAPE
1	Single Exponential Smoothing	10.00	12.56	2.90
2	Double Exponential Smoothing	3.27	6.09	1.14
3	Linear Regression	3.16	5.38	1.11
4	ARIMA(2,1,2)	2.55	4.43	0.90

From Table 6, it is seen that the best linear model is ARIMA(2,1,2) which is combined with the optimum neural network model NN(5,10,1). The model used to construct hybrid model combining the optimum neural network.

The performance measures achieved by the hybrid model using ARIMA(2,1,2) and NN(5,10,1) are shown in Table 7.

Table 7. Performance Measures for Hybrid Model

PERFORMANCE MEASURES	VALUES
MAE	0.34
MAPE	0.11
RMSE	0.18

F. Comparison of Models

Table 8 shows a comparison of MAE, MAPE and RMSE values obtained using the best linear model (ARIMA(2,1,2), Optimum Neural Network (NN(5,10,1)) and Hybrid models. For the sake of illustration, the results obtained by using the hybrid model with a nonlinear-linear combination are also shown in Table 8. In this case the NN(5,10,1) is applied first which is followed by ARIMA(2,1,2).

Table 8. Comparison of Performance Measures for Prediction Models

NO	MODELS	PERFORMANCE MEASURES		
		MAE	RMSE	MAPE
1	The Best Linear Model	2.55	4.43	0.90
2	The Optimum Neural Network	2.22	2.84	0.85
3	Hybrid (ARIMA (2,1,2) + NN(5,10,1))	0.34	0.18	0.11
4	Hybrid (NN(5,10,1) + ARIMA (2,1,2))	2.22	6.78	0.80

We note from Table 8 that the hybrid model combining ARIMA (2,1,2) and NN(5,10,1) gives the best results compared to all other models.

VI. CONCLUSION

This paper has discussed the use of an adaptive hybrid algorithm combining linear and neural network models for the prediction of time series data pertaining to morbidity of tuberculosis. This algorithm will test the linearity of the time series data to determine which hybrid combination, namely, linear-nonlinear or nonlinear-linear should be used for prediction. From the experimental results, it has been found that the hybrid model comprising linear - nonlinear combination performs better than other models for the prediction of MTB data.

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