A Joint Filtering based SVD Technique for Image De-noising

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Abstract—**Image de-noising is an important part of image enhancement. This paper proposes an image de-noising technique on the singular value decomposition (SVD) using a combination of the signal-to-noise ratio (SNR) and median of noised image as a filtering function for singular values replacement. The signal noise is assumed to be an additive white Gaussian noise. Experimental results show that the proposed method is able to successfully enhance the visual quality of noisy images through this low complexity de-noising process.**

Keywords-component; image de-noising; singular value decomposition; signal-to-noise ratio; median; image enhancement

I. INTRODUCTION

The term 'noisy image' refers to image degradation caused by noise perturbation. Image de-noising is the process of reducing or removing the influence of noise from the noisy image and reproducing, as close as possible, the original 'clean' image.

Singular value decomposition (SVD) is a technique that has been widely applied in image de-noising. Yanmin *et al.* [1] proposed an adaptive de-noising by SVD using image patches. Their experiment achieved outstanding preservation of image details, and provided improvement on de-noised images at high noise levels. Sunil and Yadava [2] proposed noise removal by truncating the SVD matrices up to a few largest singular value components, and reconstructing the de-noised image by using the remaining singular vectors. Their procedure could effectively remove an additive noise from the sensor array based electronic nose data. Tanaphol *et al.* [3] proposed an adaptive image de-noising based on the non-local mean by employing the SVD and K-means clustering technique for robust block classification in noisy images, adjusting the local window adaptively to match the local property of a block, and applying a rotated block matching algorithm for better similarity matching. Their proposed technique is shown to be effective in de-noising highly noisy images. *Zhijia et al.* [4] proposed the minimum energy model for image de-noising by selecting the proper singular values that represent the signal, and discarding the ones that represent noise. Their experiment results show that their technique is effective and robust to the images with simple/regular pattern/structure. Wongsawat *et al.* [5] proposed the multichannel SVD-based image de-noising by employing the integer discrete cosine transform (IntDCT) to de-correlate the image into sixteen sub-bands and applying the SVD to each of the subbands. Their proposed technique could effectively filter the noisy images without assuming any statistics of the image.

In this work, we propose an alternative method to reduce the noise by using the SNR and median values of noised image as a filtering function based on SVD. In SVD, singular values have a dominant effect on the image quality [4]. By modifying those values using this filtering, the proposed method could be expected to be as close as possible to the original clean image.

This paper is organized as follows. Section II illustrates the SVD in brief as a background of this work. Section III presents the proposed image de-noising method, while the experimental results are described in Section IV. Finally, the conclusion is presented in Section V.

II. BACKGROUND

A. Singular Value Decomposition

The SVD is a numerical approach to obtain a linear algebraic solution by matrix factorization. A matrix can be decomposed into three matrices of the same size as the original matrix, which in turn, can be reconstructed into the original matrix.

Let, M be the $N \times N$ real matrix with rank $r \leqslant N$. The SVD of M is defined as [4, 5, 8]:

$$M = USV^T \quad (1)$$

where S is an $N \times N$ diagonal matrix with singular values $s_1 \geqslant s_2 \geqslant \cdots \geqslant s_N \geqslant 0$; U and V are $N \times N$ orthogonal matrices and called as the left and right singular vectors, respectively; and V^T is the conjugate transpose of V. Equation (1) can also be expressed in summation form of components matrices: u_i, s_i, and v_i^T [2].

$$M = \sum_{i=1}^{r} s_i u_i v_i^T \quad (2)$$

where $s_i = s_1 \geq s_2 \geq \ldots \geq s_N > 0$ are singular values of M; $u_i = [u_{1i}\ u_{2i}\ \ldots\ u_{Ni}]$ and $v_i = [v_{1i}\ v_{2i}\ \ldots\ v_{Ni}]$ are the left and right singular vectors with i= 1, 2, …, N, respectively.

B. SVD on Image Processing

In digital image processing, an image $N \times N$ can be represented as a matrix of size $N \times N$. Fig. 1 illustrates the SVD application to decompose and reconstruct a gray level image of Lena. The Lena image I is decomposed into U, S, and V using the SVD transform. The three matrices: U, S, and V have the same size with I. The reconstructed image of I can be obtained by multiplying U, S, and V^T.

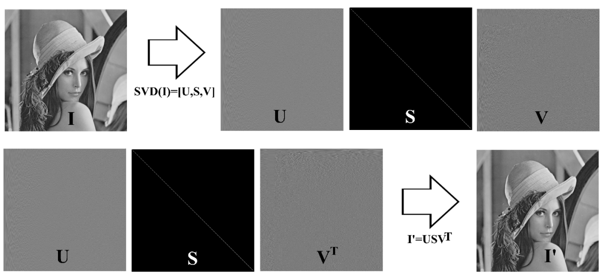

Figure 1. Image Decomposition and Reconstruction

The diagonal matrix S consists of the singular values s_i and it represents the energy of an image I [4]. It means that the image information is addressed by those singular values of S. The U and V control the spatial distribution of image energy, formulated by multiplying U and V^T as a component image. This concept is illustrated in Fig. 2.

(a) **(b)**

Figure 2. Image Spatial Distribution of Lena$_{512\times512}$ (a) Original Image, (b) UV^T Image

There are many SVD applications in image processing, such as image compression, registration, recognition, enhancement, and segmentation. The SVD transformation has some important advantages in image processing. First, the singular values of an image are stable without any great variance when the image has a small disturbance. Second, singular values contain algebraic image properties which are intrinsic and not visual [7].

III. The Proposed Image De-Noising Method

Suppose the original image I is distorted by a noise component X. The noisy image I' can be formulated as:

$$I' = I + X \tag{3}$$

where the noise variable X is a random noise and is assumed to be independent and identically distributed (*i.i.d.*) Gaussian distribution with zero mean and standard deviation σ. The main task of the image de-noising is to estimate the noiseless image, namely, I_d, from the noisy image I'.

In this work, the estimated image I_d will be achieved by modifying the singular values of I_d. Let Λ be a diagonal matrix which consists of the singular values s_i, and r be a rank of Λ. Noise reduction is performed by replacing the singular values s_i in Λ using the filtering function in (4). The change of singular values is obtained based on their certain position which is determined by the percentile values (P_k). The singular values are divided into four ranges with the limit of position values are P_5, P_{50}, and P_{75}. The most image information of an image is represented on the foreside of the singular matrix and its represent by P_5 to P_{50} of the first-half singular values. The next half range of singular values then separated proportionally. This separation is intended to prevent a significant loss of image information, especially on the first-half singular values.

$$f(s_i) = \begin{cases} s_i - \alpha \times med_\Lambda \times SNR_\Lambda^2, i < P_5 \\ s_i - \alpha \times med_\Lambda \times (1 - SNR_\Lambda^2), P_5 \leq i < P_{50} \\ s_i \times \alpha \times med_\Lambda \times SNR_\Lambda^2, P_{50} \leq i < P_{75} \\ s_i \times \alpha, i \geq P_{75} \end{cases} \tag{4}$$

where α is the de-noising coefficient, med_Λ is the median of Λ, P_k is the k-percent position (percentile) of the s_i, and i = 1, 2,…, r. The SNR_Λ is a ratio of the mean and standard deviation of Λ, notated as [4,7]:

$$SNR_\Lambda = \mu_\Lambda / \sigma_\Lambda \tag{5}$$

where μ_Λ and σ_Λ are the mean and standard deviation of Λ, respectively.

To evaluate the proposed method performance, the mean square error (MSE) and peak signal to noise ratio (PSNR) are used in this work as performance measures. The MSE and PSNR are computed by (6) and (7), respectively.

$$MSE = \frac{1}{mn} \sum_{i=1}^{m} \sum_{i=1}^{n} [I(i,j) - I'(i,j)]^2 \tag{6}$$

$$PSNR = 10.\log_{10}\left(Max_I^2 / MSE\right) \tag{7}$$

where I and I' are the original and noiseless image, respectively; Max_I is the maximum possible pixel value of the image I; and m and n are the number of rows and columns of the image I, respectively.

IV. Experimental Results

The noise model in this experiment is assumed to be additive Gaussian with zero mean and standard deviation σ. For the first experiment, we used standard deviation σ=10 to generate a random noise and α=0.25 as a de-noising coefficient. The noisy and de-noised images of Lena$_{512\times512}$ are illustrated in Fig. 3a and Fig. 3b. Using those parameter values, the MSE is decreased by 87.5803, while the PSNR is increased by 9.0666dB. It means that the proposed method could improve the image visual quality by reducing the noise.

(a) (b)

Figure 3. Noisy and De-noised Image of Lena$_{512\times512}$. (a) Noisy Image (σ=10, PSNR=28.1319dB, MSE=99.9748), (b) De-noised Image (α=0.25, PSNR=37.1985dB, MSE=12.3945

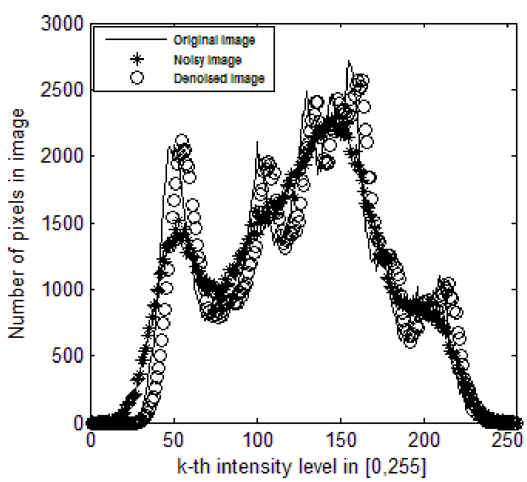

Fig. 4. Plot Histogram of Lena$_{512\times512}$

To evaluate the estimation model, Fig. 4 illustrates the histogram of the Lena image for standard deviation σ=10. The change on the singular values of the noisy image reduces the noise when the plot gets closer to that of the original image. Although the de-noised image is not a perfect estimation of the clean image, the pixel distribution of the de-noised image is significantly similar to the original image. The residual image which represents the differences between the original image and de-noised image is illustrated in Fig. 5.

The proposed method was then applied on test images with higher standard deviation of noise. The standard test images, such as Lena, Pepper and Goldhill were used in the performance evaluation. The noisy, de-noised, and histogram images of Pepper and Goldhill are illustrated in Fig. 6. The MSE and PSNR performances are presented in Table I. The standard deviations in this experiment are 10, 15, 20, and 25; with the fixed de-noised coefficient is 0.25. Experimental results show that using the higher standard deviation, the MSE increased and the PSNR decreased.

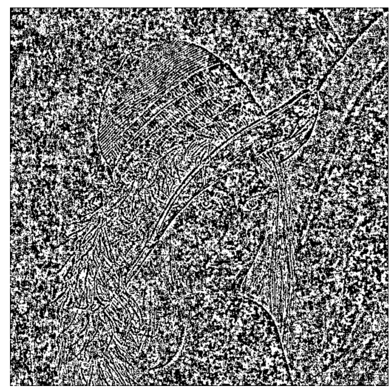

Fig. 5. Residual Image of Lena$_{512\times512}$

TABLE I. MSE And PSNR Performances in Different Images (α=0.25)

Images	σ	MSE		PSNR (dB)	
		Noisy	De-noised	Noisy	De-noised
Lena	10	99.9748	12.3945	28.1319	37.1985
	15	224.487	20.4109	24.6189	35.0322
	20	400.673	31.8340	22.1029	33.1019
	25	622.774	47.0392	20.1875	31.4062
Pepper	10	99.5635	11.3899	28.1498	37.5656
	15	225.004	19.2252	24.6089	35.2921
	20	400.553	30.2503	22.1042	33.3235
	25	626.672	45.0174	20.1604	31.5970
Goldhill	10	99.5635	22.0653	28.1498	34.6937
	15	224.864	30.6521	24.6116	33.2662
	20	400.766	42.4503	22.1019	31.8520
	25	622.286	57.3285	20.1909	30.5471

The performance of the proposed method is also evaluated for different values of the de-noising coefficient α. Table II shows that the change of the de-noising coefficient caused the small effect on the visual quality of performance both in MSE and PSNR. Similar results were observed.

TABLE II. MSE AND PSNR PERFORMANCES OF LENA IMAGE IN DIFFERENT DE-NOISING COEFFICIENT (σ =10)

α	σ	MSE		PSNR (dB)	
		Noisy	De-noised	Noisy	De-noised
0.00	10	99.6392	12.7462	28.1465	37.0770
0.25	10	99.6966	12.4724	28.1440	37.1713
0.50	10	99.9425	12.3982	28.1333	37.1972
0.75	10	99.7127	12.5061	28.1433	37.1596
1.00	10	100.1637	12.7450	28.1237	37.0774

In evaluation of the proposed method with the other benchmark methods, the results presented in Table III show that the proposed method performed better in almost all cases.

TABLE III. PSNR PERFORMANCES COMPARISON

Images	σ	MSVD [5]	ANL [3]	ASVD [1]	Proposed α=1
Lena	10	32.12	34.11	35.60	37.06
	20	28.56	31.98	32.97	33.64
	25	27.45	30.56	32.05	32.19
	30	26.67	30.04	31.13	30.88
	40	25.79	28.27	29.82	28.73
	50	24.38	27.29	28.94	26.79
Pepper	10	31.68	34.65	36.19	37.57
	20	28.48	31.59	32.84	33.98
	25	27.75	30.21	31.43	32.49
	30	26.12	29.79	30.98	31.16
	40	25.04	28.00	29.13	28.80
	50	24.19	27.12	28.43	26.95
Goldhill	10	30.83	31.67	32.84	34.69
	20	27.69	30.15	30.42	32.40
	25	27.04	29.12	29.60	31.28
	30	26.11	28.28	28.81	30.21
	40	25.53	27.98	28.42	28.26
	50	24.77	26.56	27.13	26.52

V. CONCLUSION

This paper proposed an image de-noising technique using singular value decomposition (SVD) by median and SNR as a filtering function. To obtain the clean signal, the filtering was applied to change the singular values. In this method, the low singular values were kept to prevent drastic changes to those values. Experimental work demonstrates that the proposed method has better performance in reducing most noise compared to the other methods. As a further work, the proposed method will be applied on the block-image.

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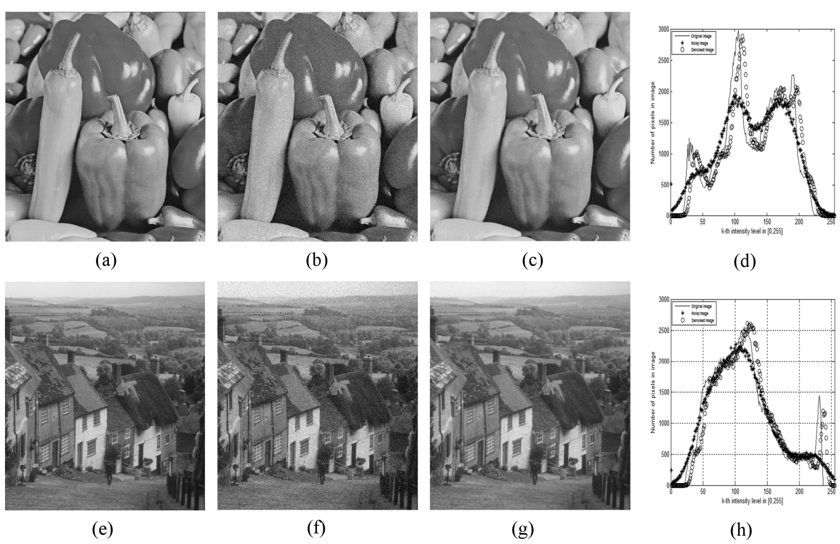

Fig. 6. (a)-(d) Original, Noisy, De-Noised, Histogram Image of Pepper$_{512\times512}$;(e)-(h) Original, Noisy, De-Noised, Histogram Image of GoldHill$_{512\times512}$